8. (i) By (10.21) and (10.20) with $c = 1$,

$$u(x,t) = \int_0^t \int_{-\infty}^{\infty} G(x,t;\xi,\tau) q(\xi,\tau)\, d\xi\, d\tau$$

$$= \int_0^t \int_{-\infty}^{\infty} \tfrac{1}{2}\big[H(x-\xi+t-\tau) - H(x-\xi-t+\tau)\big] q(\xi,\tau)\, d\xi\, d\tau$$

$$= \frac{1}{2}\int_0^t \int_{-\infty}^{x+t-\tau} q(\xi,\tau)\, d\xi\, d\tau - \frac{1}{2}\int_0^t \int_{-\infty}^{x-t+\tau} q(\xi,\tau)\, d\xi\, d\tau = \frac{1}{2}\int_0^t \int_{x-t+\tau}^{x+t-\tau} q(\xi,\tau)\, d\xi\, d\tau$$

$$\Rightarrow \quad u(-1,2) = \frac{1}{2}\int_0^2 \int_{-3+\tau}^{1-\tau} q(\xi,\tau)\, d\xi\, d\tau,$$

$$0 < \tau < 2 \quad \Rightarrow \quad -3+\tau < -1 < 1-\tau < 1$$

$$\Rightarrow \quad u(-1,2) = \frac{1}{2}\int_0^2 \int_{-1}^{1-\tau} \tau\, d\xi\, d\tau = \frac{1}{2}\int_0^2 \tau(2-\tau)\, d\tau = \tfrac{1}{2}\big[\tau^2 - \tfrac{1}{3}\tau^3\big]_0^2 = \tfrac{2}{3},$$

$$u(0,2) = \frac{1}{2}\int_0^2 \int_{-2+\tau}^{2-\tau} q(\xi,\tau)\, d\xi\, d\tau,$$

$$0 < \tau < 1 \quad \Rightarrow \quad -2+\tau < -1, \quad 2-\tau > 1,$$
$$1 < \tau < 2 \quad \Rightarrow \quad -2+\tau > -1, \quad 2-\tau < 1$$

$$\Rightarrow \quad u(0,2) = \frac{1}{2}\int_0^1 \int_{-1}^{1} \tau\, d\xi\, d\tau + \frac{1}{2}\int_1^2 \int_{-2+\tau}^{2-\tau} \tau\, d\xi\, d\tau$$

$$= \frac{1}{2}\int_0^1 2\tau\, d\tau + \frac{1}{2}\int_1^2 \tau(4-2\tau)\, d\xi = \tfrac{1}{2}\big[\tau^2\big]_0^1 + \big[\tau^2 - \tfrac{1}{3}\tau^3\big]_1^2 = \tfrac{7}{6},$$

$$u(2,3) = \frac{1}{2}\int_0^3 \int_{-1+\tau}^{5-\tau} q(\xi,\tau)\, d\xi\, d\tau,$$

$$0 < \tau < 2 \quad \Rightarrow \quad -1 < -1+\tau < 1, \quad 5-\tau > 1,$$
$$2 < \tau < 3 \quad \Rightarrow \quad -1+\tau > 1$$

$$\Rightarrow \quad u(2,3) = \frac{1}{2}\int_0^2 \int_{-1+\tau}^{1} \tau \, d\xi \, d\tau = \frac{1}{2}\int_0^2 \tau(2-\tau)\, d\tau = \tfrac{2}{3}.$$

(ii) As in (i) but with $c = 2$,

$$\begin{aligned}
u(x,t) &= \int_0^t \int_{-\infty}^{\infty} G(x,t;\xi,\tau) q(\xi,\tau)\, d\xi \, d\tau \\
&= \int_0^t \int_{-\infty}^{\infty} \tfrac{1}{4}\big[H\big((x-\xi)+2(t-\tau)\big) - H\big((x-\xi)-2(t-\tau)\big)\big] q(\xi,\tau)\, d\xi \, d\tau \\
&= \frac{1}{4}\int_0^t \int_{-\infty}^{x+2(t-\tau)} q(\xi,\tau)\, d\xi\, d\tau - \frac{1}{4}\int_0^t \int_{-\infty}^{x-2(t-\tau)} q(\xi,\tau)\, d\xi\, d\tau \\
&= \frac{1}{4}\int_0^t \int_{x-2(t-\tau)}^{x+2(t-\tau)} q(\xi,\tau)\, d\xi\, d\tau
\end{aligned}$$

$$\Rightarrow \quad u(-2,1) = \frac{1}{4}\int_0^1 \int_{-4+2\tau}^{-2\tau} q(\xi,\tau)\, d\xi\, d\tau,$$

$$0 < \tau < \tfrac{1}{2} \quad \Rightarrow \quad -4+2\tau < -1, \quad -1 < -2\tau < 0,$$
$$\tfrac{1}{2} < \tau < 1 \quad \Rightarrow \quad -2\tau < -1$$

$$\Rightarrow \quad u(-2,1) = \frac{1}{4}\int_0^{1/2} \int_{-1}^{-2\tau} \xi \, d\xi\, d\tau = \frac{1}{4}\int_0^{1/2} \tfrac{1}{2}\left[\xi^2\right]_{-1}^{-2\tau} d\tau$$

$$= \frac{1}{4}\int_0^{1/2} (2\tau^2 - \tfrac{1}{2})\, d\tau = \tfrac{1}{4}\left[\tfrac{2}{3}\tau^3 - \tfrac{1}{2}\tau\right]_0^{1/2} = -\tfrac{1}{24},$$

$$u(1,1) = \frac{1}{4}\int_0^1 \int_{-1+2\tau}^{3-2\tau} q(\xi,\tau)\, d\xi\, d\tau,$$

$$0 < \tau < 1 \quad \Rightarrow \quad -1 < -1+2\tau < 1, \quad 3-2\tau > 1$$

$$\Rightarrow \quad u(1,1) = \frac{1}{4}\int_0^1 \int_{-1+2\tau}^{1} \xi \, d\xi \, d\tau = \frac{1}{4}\int_0^1 \tfrac{1}{2}\left[\xi^2\right]_{-1+2\tau}^{1} d\tau$$

$$= \frac{1}{8}\int_0^1 \left[1-(-1+2\tau)^2\right] d\tau = \frac{1}{8}\int_0^1 (4\tau - 4\tau^2)\, d\tau$$

$$= \tfrac{1}{8}\left[2\tau^2 - \tfrac{4}{3}\,\tau^3\right]_0^1 = \tfrac{1}{12},$$

$$u(2,4) = \frac{1}{4}\int_0^4 \int_{-6+2\tau}^{10-2\tau} q(\xi,\tau)\, d\xi \, d\tau,$$

$$0 < \tau < \tfrac{5}{2} \quad \Rightarrow \quad -6+2\tau < -1, \quad 10-2\tau > 1,$$

$$\tfrac{5}{2} < \tau < \tfrac{7}{2} \quad \Rightarrow \quad -1 < -6+2\tau < 1, \quad 10-2\tau > 1,$$

$$\tfrac{7}{2} < \tau < 4 \quad \Rightarrow \quad -6+2\tau > 1$$

$$\Rightarrow \quad u(2,4) = \frac{1}{4}\int_0^{5/2}\int_{-1}^{1} \xi \, d\xi \, d\tau + \frac{1}{4}\int_{5/2}^{7/2}\int_{-6+2\tau}^{1} \xi \, d\xi \, d\tau$$

$$= \frac{1}{4}\int_0^{5/2} \tfrac{1}{2}\left[\xi^2\right]_{-1}^{1} d\tau + \frac{1}{4}\int_{5/2}^{7/2} \tfrac{1}{2}\left[\xi^2\right]_{-6+2\tau}^{1} d\tau = \frac{1}{8}\int_{5/2}^{7/2} \left[1-(-6+2\tau)^2\right] d\tau$$

$$= \frac{1}{8}\int_{5/2}^{7/2} (-35 + 24\tau - 4\tau^2)\, d\tau = \tfrac{1}{8}\left[-35\tau + 12\tau^2 - \tfrac{4}{3}\,\tau^3\right]_{5/2}^{7/2} = \tfrac{1}{12}.$$